# Supplementary File C: Extension of the reaction norm approach to incorporate 1 genetic factor and 2 ecological factors

I here show how the reaction norm approach can be extended to include one genetic and two environmental factors. Consider a change in a phenotypic trait $z$ of a population associated with a change in the environment given by two separate variables. For example, a change in the environment from low to high temperature and from low to high nutrient availability. This means that we could characterise the population by one genetic state and two ecological factors (e.g. temperature and nutrients). Denote the genetic state as an indicator variable $\tilde{g}$ that equals 0 (resp. 1) to represent the genetic make-up of the population at time point $t_1$ (resp. $t_2$), and denote with $\tilde{k}_1$ and $\tilde{k}_2$ the indicator variables representing the two ecological factors that each equal 0 (resp. 1) to represent the ecological state at time point $t_1$ (resp. $t_2$). The trait change $\Delta z$ between the two time points equals: $\Delta z = z_{222} - z_{111}$, where $z_{ijk}$ is the average value for $z$ at genetic state $i$, and ecological state $jk$, where $j$ (resp. $k$) is the index referring to the first (resp. second) ecological factor. Both the formulae of the method from Ellner et al. (2011) and the reaction norm approach can be found by solving the least-squares normal equations of a linear regression model either without or with an interaction component included (Supplementary File B). Therefore, we use a similar technique to find the formulae for the reaction norm approach having one genetic and two ecological states. We thus solve the least-squares normal equation of a linear regression model with interaction components where $z$ is the response variable and $\tilde{g}$, $\tilde{k}_1$, and $\tilde{k}_2$ are used as predictor variables, i.e.

$$
\begin{aligned}
z = \alpha + \beta_g \tilde{g} + \beta_{k_1} \tilde{k}_1 + \beta_{k_2} \tilde{k}_2 + \gamma_{g \times k_1} \tilde{g} \times \tilde{k}_1 + \gamma_{g \times k_2} \tilde{g} \times \tilde{k}_2 \\
+ \gamma_{k_1 \times k_2} \tilde{k}_1 \times \tilde{k}_2 + \gamma_{g \times k_1 \times \tilde{k}_2} \tilde{g} \times \tilde{k}_1 \times \tilde{k}_2 + error
\end{aligned}
\tag{C.1}
$$

$\alpha$ represents the intercept, and $\beta$ and $\gamma$ represent the regression coefficients of the main and interaction effects of $\tilde{g}$, $\tilde{k}_1$ and $\tilde{k}_2$. Solving the least-squares normal equations for the model coefficients allows to partition the change in $\Delta z = z_{222} - z_{111}$ into the following effects:

$$
\begin{aligned}
g &= z_{211} - z_{111} \\
k_2 &= z_{121} - z_{111} \\
k_2 &= z_{112} - z_{111} \\
g \times k_1 &= [z_{221} - z_{211}] - [z_{121} - z_{111}] \\
g \times k_2 &= [z_{212} - z_{211}] - [z_{112} - z_{111}] \\
k_2 \times k_2 &= [z_{122} - z_{121}] - [z_{112} - z_{111}] \\
g \times k_2 \times k_2 &= ([z_{\mathbf{22}2} - z_{\mathbf{21}2}] - [z_{\mathbf{12}2} - z_{\mathbf{11}2}]) - ([z_{\mathbf{22}1} - z_{\mathbf{21}1}] - [z_{\mathbf{12}1} - z_{\mathbf{11}1}]) \\
&= ([z_{\mathbf{222}} - z_{\mathbf{221}}] - [z_{\mathbf{122}} - z_{\mathbf{121}}]) - ([z_{\mathbf{212}} - z_{\mathbf{211}}] - [z_{\mathbf{112}} - z_{\mathbf{111}}])
\end{aligned}
\tag{C.2}
$$

The first line in (C.2) gives the change in genetic states at the ecological states of time point $t_1$. The second and third line give the change in ecological state of either the first ($k_1$)

or second ($k_2$) ecological factor at the genetic state of time point $t_1$. This is similar as the plasticity component of the population at the first time point, and might thus reflect an ancestral plasticity response (Stoks et al. 2016; Govaert et al. 2016). $g \times k_1$ and $g \times k_1$ reflect the change in plasticity response to either $k_1$ and $k_2$ between the population at time point $t_1$ and $t_2$, and can be interpreted similar to the evolution of plasticity component found in the reaction norm approach described in Stoks et al. (2016) and Govaert et al. (2016). The sixth line in (C.2) gives the change in $z$ due to a simultaneous change in both ecological factors. The three-way interaction gives the change in $z$ due to simultaneous changes in genetic states with ecological states. This might reflect the change in evolution of plasticity (indexed in bold) of either $k_1$ (seventh line) or $k_2$ (eighth line) depending on the change in the ecological state of either $k_2$ or $k_1$.

# References


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