Supplementary File B: Derivation of the method of Ellner et al. (2011) and the reaction norm approach

The reaction norm approach and the method of Ellner et al. (2011) both require the construction of reaction norms, this means that one should have data available on the average trait values of a population at two time points differing in a particular environmental condition (e.g. presence versus absence of a predator) in the two environmental conditions. The method of Ellner et al. (2011) (eqn (2) in main text) was originally developed to disentangle change in an ecological response variable (given by X) into an ecological (via an ecological or environmental factor) and evolutionary (via the genetic part of phenotypic trait) component, while the reaction norm approach (eqn (3) in main text) was directly aimed at disentangling the change in the phenotypic trait z itself. However, both methods can be applied to assess ecological and evolutionary contributions to either trait change itself or to an ecological response variable linked to an ecological factor and phenotypic trait. For example, terHorst et al. (2014) and Pantel et al. (2015), using the method of Ellner et al. (2011), directly measured values of their ecological response variables for all combinations of X. Substituting experimentally taken measurements into the method of Ellner et al. (2011) allows direct assessment of main effects of ecology and evolution to the observed change in X, without needing to the link the ecological factor and phenotypic trait to the ecological response variable. Other studies, such as Becks et al. (2012) and Pigéon et al. (2017), first construct a link between their ecological response variable and a phenotypic trait influencing that response variable by using a regression model. This regression model then makes it possible to estimate the different combinations of the ecological response variable X, i.e. all X_{ij}.

The formulae of both methods can be derived by solving the least-squares normal equations for the model coefficients of a linear regression model either with an interaction term for the reaction norm approach or without an interaction term for the method of Ellner et al. (2011). As described in Ellner et al. (2011), but briefly repeated here for illustrative reasons. Consider a change from a genetic and ecological state 1 at time t_1, to a genetic and ecological state 2 at time t_2. Define the genetic and ecological state with an indicator variable $\tilde{g}$ and $\tilde{k}$ that equals 0 (resp. 1) for values corresponding to time point t_1 (resp. time point t_2). To obtain the formulae given in eqn (12) in Ellner et al. (2011), given by eqn (2) in the main text, but for a phenotypic trait z directly, we solve the least-squares normal equations for the model coefficients of the following linear regression model:

$$z = \alpha + \beta_g \tilde{g} + \beta_k \tilde{k} + \epsilon, \tag{B.1}$$

with α the intercept, β_g and β_k the regression coefficients of the indicator variable $\tilde{g}$ and $\tilde{k}$ and ϵ the error term. The model coefficients $\beta = \mathbf{C}z$ where $\beta = (\beta_g, \beta_k)$, $\mathbf{C} = (\mathbf{Y}^T\mathbf{Y})^{-1}\mathbf{Y}^T$, and $z = (z_{11}, z_{12}, z_{21}, z_{22})$ is the data vector. Moreover, matrix $\mathbf{Y}$ is a (4×3)-matrix that equals:

$$\mathbf{Y} = \begin{array}{c} \\ z_{11} \\ z_{21} \\ z_{12} \\ z_{22} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{intercept} & \tilde{g} & \tilde{k} \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \end{array}. \tag{B.2}$$

C then gives the formula for the main effect of plasticity and evolution as:

$$\mathbf{C} = \begin{array}{c} \\ \textit{intercept} \\ \tilde{g} \\ \tilde{k} \end{array} \begin{array}{c} \begin{array}{cccc} z_{11} & z_{21} & z_{12} & z_{22} \end{array} \\ \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ -0.50 & 0.50 & -0.50 & 0.50 \\ -0.50 & -0.50 & 0.50 & 0.50 \end{pmatrix} \end{array}$$

Phenotypic trait change $\Delta\bar{z}$ can then be decomposed into main effects of plasticity and evolutionary change as follows:

$$\bar{z} = \frac{1}{2}\Big[(z_{12} - z_{11}) + (z_{22} - z_{21})\Big] + \frac{1}{2}\Big[(z_{21} - z_{11}) + (z_{22} - z_{12})\Big], \tag{B.3}$$

where z_{kl} equals the trait mean of the population at time point t_k at environmental condition l. On the other hand, the formulae for the reaction norm approach are obtained by solving the least-squares normal equations for the model coefficients of a linear regression model with interaction component. Consider the same indicator variables $\tilde{g}$ and $\tilde{k}$ just described, then the linear regression model with interaction term is given as follows:

$$z = \alpha + \beta_g \tilde{g} + \beta_k \tilde{k} + \gamma \tilde{g} \times \tilde{k} + \epsilon. \tag{B.4}$$

Similarly, by solving the least-squares normal equations of this model, we obtain the regression coefficients $\beta = (\beta_g, \beta_k, \gamma)$ where $\beta = \mathbf{C}z$ with $\mathbf{C} = (\mathbf{Y}^T\mathbf{Y})^{-1}\mathbf{Y}^T$. Here, matrix $\mathbf{Y}$ is a (4×4) matrix, i.e.

$$\mathbf{Y} = \begin{array}{c} \\ z_{11} \\ z_{21} \\ z_{12} \\ z_{22} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{intercept} & \tilde{g} & \tilde{k} & \tilde{g} \times \tilde{k} \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \end{array}. \tag{B.5}$$

C then equals:

$$\mathbf{C} = \begin{array}{c} \\ \textit{intercept} \\ \tilde{g} \\ \tilde{k} \\ \tilde{g} \times \tilde{k} \end{array} \begin{array}{c} \begin{array}{cccc} z_{11} & z_{21} & z_{12} & z_{22} \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 \end{pmatrix} \end{array} \tag{B.6}$$

This thus yields the formula as given by the reaction norm approach, and partitions observed trait change into plasticity, constitutive evolution and evolution of plasticity, i.e.

$$\bar{z} = (z_{12} - z_{11}) + (z_{21} - z_{11}) + \Big([z_{22} - z_{21}] - [z_{12} - z_{11}]\Big) \tag{B.7}$$

However, the method of Ellner et al. (2011) can also include an interaction term. Including an interaction term and straightforwardly solving the least-squares normal equations of a linear regression model with interaction component would result in obtaining the formulae of the reaction norm approach. In order to obtain the main effects of evolution and ecology as given in the method of Ellner et al. (2011), one should set the contrasts to Helmert contrast before solving the least-squares normal equations for the model coefficients of a linear regression model with interaction term (Pantel et al. 2015). Using Helmert contrasts moves the intercept to the midpoint (i.e. average value among all combinations), and therefore compares each level of a categorical variable to the mean of subsequent levels. The setting of these contrasts explains why the interpretation of the components in the equation of Ellner et al. (2011) can be seen as 'average' main effects. With Helmert contrasts, matrix **Y** given in (B.5) instead becomes:

$$\mathbf{Y} = \begin{array}{c} \\ z_{11} \\ z_{21} \\ z_{12} \\ z_{22} \end{array} \begin{array}{c} \begin{array}{cccc} intercept & \tilde{g} & \tilde{k} & \tilde{g} \times \tilde{k} \end{array} \\ \begin{pmatrix} 1 & -0.5 & -0.5 & 0.5 \\ 1 & 0.5 & -0.5 & -0.5 \\ 1 & -0.5 & 0.5 & -0.5 \\ 1 & 0.5 & 0.5 & 0.5 \end{pmatrix} \end{array} \tag{B.8}$$

and **C** solves to:

$$\mathbf{C} = \begin{array}{c} \\ intercept \\ \tilde{g} \\ \tilde{k} \\ \tilde{g} \times \tilde{k} \end{array} \begin{array}{c} \begin{array}{cccc} z_{11} & z_{21} & z_{12} & z_{22} \end{array} \\ \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ -0.50 & 0.50 & -0.50 & 0.50 \\ -0.50 & -0.50 & 0.50 & 0.50 \\ 0.50 & -0.50 & -0.50 & 0.50 \end{pmatrix} \end{array} \tag{B.9}$$

Thus the interaction component here equals

$$G \times K = \frac{1}{2}\Big[(z_{22} - z_{21}) - (z_{12} - z_{11})\Big] \tag{B.10}$$

which has the extra division by 2 compared with the interaction component of the reaction norm approach.

The formula for the intercept reflects the comparison level of the two methods, and this difference in comparison level influences the interpretation of the components. The reaction norm approach calculates the components compared with the ancestral population, i.e. the oldest time point, as opposed to a midpoint. This is reflected in the plasticity component referring to the ancestral plasticity, and evolution of plasticity being the evolved plasticity response compared with the ancestral plasticity response. The latter resulting in an intuitive interpretation of the eco-evolutionary interaction component of the reaction norm approach. While in the method of Ellner et al. (2011) the interaction component is similar to the formula of evolution of plasticity as given in the reaction norm approach (except for the division by 2), it is less clear if these mathematically equivalent terms reflect the same process. Moreover, the main effects of plasticity and evolution in the method of Ellner et al. (2011) add up to the change in z (i.e. $\Delta\bar{z}$). Incorporating the eco-evolutionary interaction component in this method leads to an extra component that does not add up to the main effects to give the change in z.

References

[1] Ellner, S. P., Geber, M. A., and Hairston, N. G. (2011). Does rapid evolution matter? Measuring the rate of contemporary evolution and its impacts on ecological dynamics. *Ecology Letters 14*, 6, 603–614. doi:10.1111/j.1461-0248.2011.01616.x.

[2] Pantel, J. H., Duvivier, C., and De Meester, L. (2015). Rapid local adaptation mediates zooplankton community assembly in experimental mesocosms. *Ecology Letters 18*, 10, 992–1000. doi:10.1111/ele.12480.

[3] Pigéon, G., Ezard, T. H., Festa-Bianchet, M., Coltman, D. W., and Pelletier, F. (2017). Fluctuating effects of genetic and plastic changes in body mass on population dynamics in a large herbivore. *Ecology 98*, 9, 1–40. doi:10.1002/ecy.1940.

[4] terHorst, C. P., Lennon, J. T., and Lau, J. A. (2014). The relative importance of rapid evolution for plant-microbe interactions depends on ecological context. *Proceedings of the Royal Society of London B: Biological Sciences 281*, 1785, 20140028. doi:10.1098/rspb.2014.0028.